

Syntactic and Semantic Distribution in Quantum Measurement

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Abstract

The nondistributivity of compound quantum mechanical propositions leads to a theorem that rules out the possibility of microscopic deterministic hidden variables, the Logical No-Go Theorem. We observe that there appear in fact two distinct nondistributivity relations in the derivation: one with a semantics governed by an empirical conjunctive syntax, the other composed of conjunctive primitives in the quantum mechanical probability calculus. We venture to speculate how the two come to be confused in the derivation of the theorem.

1 introduction

The 20th century witnessed a revolution in experimental instrumentation from the likes of the Plank's black box apparatus to the Stern-Gerlach spin analyzer. From these there came a wealth of new and unusual data, much of which suggested a microscopic substructure [1] whose workings were not governed by the then prevailing Newtonian mechanics. Eventually, particles came to be seen no longer as entities with categorical properties but as carriers of properties that could only be inferred from the experimental probabilities that they collectively generate. In what has become the orthodox interpretation of the data and governing theory, quantum mechanics (QM), the reasoning is taken so far as to call into question the very notion causality implicit to scientific reasoning and as such continues to present to the interested student an array of counterintuitive conceptual challenges.

To elaborate the new conception there has over the years come several formalizations of the quantum theory ¹ whose profusion and variety however it now seems may well have had much of an opposite effect. But a great many of these may be understood as partial interpretations of the original consistent mathematical formulation credited to Von Neumann [3] and Dirac, and whose modern version is now standard to most QM texts; to understand this formulation then is to understand the foundation upon which many of the others are built.

A distinctive feature of the von Neumann formal machinery is its axiomatization of indeterminacy as fundamental to microscopic events. This is posited via the Collapse Postulate [4], which helps account for the ubiquitous dispersion of ensemble experimental values on one hand [5] and for the observation of definite individual experimental outcomes on the other. Another distinctive feature is its representation

¹Schrodinger's wave mechanics, Heisenberg's matrix mechanics, Dirac and von Neumann's Hilbert space formulation, Feynman's path integral formulation, von Neumann and Segal's c -algebra formalism, Everett's many worlds interpretation, Gell-Mann and Griffith's consistent histories formulation, quantum logic formulation advanced by von Neumann and Mackey, and others [2]

of the experimental process by the action of Hilbert space operators that among themselves generally do not commute; this to account for the observation that pairs of consecutive measurements performed on a single system when temporally reversed generally do not yield the same pair of outcomes, i.e., for the observation that such measurements also do not commute. Around these core ideas has developed an increasingly abstract semantics - rules that lay down the correspondence between the theoretical terms in the mathematical machinery of quantum mechanics and observation - now a source of conceptual difficulty for and disagreement among all interested parties from physicist to philosopher. E.g., what to a Bayesian inclined mathematician or philosopher are relations concerning the uncertainty of individual experimental outcomes [6], to an empirically minded experimentalist may be nothing more than unusual scatter relations [7]. And so forth.

This particular example highlights the central question of concern to an interpretation of the theory: Whether it is possible to supplement the quantum mechanical description of reality with additional parameters, so called hidden variables (hv), which would then together give a more 'complete' account of microscopic processes and states, including absent in the existing theory, such as those states that correspond to noncommuting experimental outcomes. On this issue there is certainly a wide range of possible views, but the leading majority opinions, as a matter of fact and history, are and have been polarized. In the affirmative view, whose early proponents include A. Einstein, the proposed notion of the quantum particle is at odds with the very concept of 'particle' conceived classically as a point in phase space, and the incompleteness of the theory is self-evident. Those in opposition, proponents of the conventional or orthodox interpretation, have gone so far as to produce explicit proofs against the very possibility, somehow managing to prove a negative.

Among these proof, popularly known as 'no-go' theorems, perhaps the best known is the one due exclusively to John Bell. By exploiting the locality requirements of special relativity Bell derives an explicit disagreement between the tenants of the

local realism and the predictions of QM, summarized in his elegant inequality [8]. Next in order of the interest it has generated over the years is the theorem of Kochen and Specker (KS) who begin by taking the possibility of isomorphisms from the Hilbert subspaces of QM to classical Boolean subspaces as a basic constraint on realist interpretations, then demonstrates that there are none. The significance of each of these is addressed by the writer in earlier works [9, 10]. Finally there is the lesser known argument against hidden variables advanced in the mid-sixties by Jauch and Piron, the Logical no-go theorem [14]. Interest in this proof however peaked and quickly declined until it is today not much discussed at all ², the remaining interest lying mainly in its close association with an earlier and similar argument by von Neumann and with the later work of Kochen and Specker. More importantly still is its place in the historical development of logical formulations of the theory, of quantum logics.

To briefly outline the basic quantum logic idea, to every experimental outcome there corresponds a proposition (for outcome 'a', the proposition: 'the experimental outcome is a'). Then the indeterminacy of measurement outcomes as axiomatized by von Neumann imposes in an obvious way a certain non-bivalence upon the truth values of the individual propositions corresponding to those outcomes (such that all experimental propositions in respect of the physical system upon which measurements are to be taken, experiments performed, are not of necessity either true or false [15]), i.e. upon the truth values of individual propositions, their system then corresponding to the set of all measurements that may be made upon the given physical system. Thus, structural features inherent to the standard formulation's Hilbert space, whose operators are bijective [11] in respect of possible observations, correspond directly to those of the proposition system. By means of semantical rules, these in turn correspond, presumably, to logical structures extant in the microscopic physical world and so now framed in the language of quantum mechanics.

²Thompson's ISI Web of Knowledge lists 1934 citations of Bell's theorem, 381 of the theorem of KS, and 92 of the logical no-go theorem, only 10 of those since 2000.

But such a system and a logic, like their Hilbert space description, are non-Boolean, hence non-classical.

Those familiar with quantum theory will probably have first encountered this distinction in some form or other of Bohr's complementarity [12], as complementary variables are also variables that do not commute; hence, the logic of their observation or measurement is necessarily non-Boolean. Given the recent important experimental *welcher-weg* tests conducted by S. Afshar and students [13] and the questions concerning complementarity raised by their results, still under review ³, a critical review of the complementary semantics, such as the present one, that also maintains an elementary presentation, could hardly seem to us more timely.

In this article we analyze the particular Logical argument against the existence of hv's put forward by Jauch and Piron [14], whose driving force we trace to a semantical rule for the conjunction of propositions, $a \cap b$, associated with pairs of measurements that do not commute, $[a, b] \neq 0$. While the set theoretic and ordinary logical semantics of the conjunction are well known, the compound being true when each proposition is separately true, in the new logic there remain questions. The syntactic structure has been analyzed over the years by many workers in the field, and in the view of some [16, 14, 17, 18, 3, 19], *prima facie* in line with its Hilbert space correspondence, as an experimental proposition the noncommutative conjunction is tautologically false. However according to others [20, 21, 22, 23, 24], and in line with more direct semantics, such compounds are not experimental propositions that bear on *individual systems* at all, but are in this respect formal expressions having no real meaning. It is entirely possible that this issue cannot be settled objectively, as the difference in opinion may be grounded in the much longer standing difference in interpretation of the quantum theory itself. In this article we attempt to understand the noncommutative conjunction exclusively in terms of its use.

In section 1 we consider a microscopic experiment instrumental in motivating the conceptual development of quantum theory and trace the noncommutative conjunc-

³For a rebuttal see W. Unruh's article at URL = \langle <http://axion.physics.ubc.ca/rebel.html> \rangle .

tive etymology within its logic to the sigma algebra of its Hilbert $H \cup H'$ probability space whose nondistributive syntax reveals the dispersive semantics that leads to the conclusions of the logical no-go theorem:

$$\begin{aligned} \text{realist interpretation} &\Rightarrow \text{value-definiteness} \Rightarrow \text{dispersion-free mixtures} \Rightarrow \\ &\text{distributive logic} \Rightarrow \text{commutative logic} \Rightarrow \text{classical physics.} \end{aligned}$$

We follow up in the next section with an examination of the probability space of compound noncommuting observations where we find a formally identical nondistributivity relation which, in contrast to the previous relation, is grounded in the metalanguage of the theory. There, the noncommuting 'conjunction' appears as an elementary or atomic event in the product $H \times H'$ space [25]. In light of the apparent distinct events they reference, we consider in the next section whether the two relations might in fact correspond to the same physical property. We find that they do not (at least from the relevant realist point of view), which then invalidates the logical no-go theorem. And while both Bell and Bohn challenged this validity long ago, their results were generally not well received at the time.

In addition to our adherence to an elementary exposition remaining within the purview of undergraduate QM, another difference between the earlier analysis and our approach is the region of analytic validity that we concede to the theorem; we observe that the opposing views operate on distinct semantics that follow, in one case, from the syntactic reduction of diatomic compound bivalent experimental propositions, and in the other, from that of such propositions over their aggregates (which are generally nonbivalent), then combined. This point of view offers we think a more comprehensive understanding of the disagreement. We end in the final section with a few concluding remarks on the theorem and related issues.

While the concepts central to the logical no-go theorem and quantum logic generally are fundamentally simple, they do involve a myriad of definitions and notations unfamiliar to most students and non-specialists, although again, no single one of these particularly difficult to grasp. It is also likely that many readers will first

encounter this article via an internet resource. For these reasons we make extensive use of internet citations. We often point to Wolfram's *MathWorld and the Statistics Glossary* for clarifications and basic definitions in probability theory, and to the *The Stanford Encyclopedia of Philosophy*, *The Philosophy Pages*, and *Wikipedia, the free encyclopedia* for philosophical and historical contexts.

2 argument against hidden variables

With an aim to predict and finally manipulate physical events and processes, the scientific enterprise proceeds on the implicit premise that given the relevant physical laws and prevailing conditions, the occurrence of subsequent events may in principle always be known beforehand. I.e., it proceeds on the premise that such physical laws indeed exist and is thus fundamentally entrenched in the determinism hypothesis. It is an irony then that the facilitating scientific method, famously successful in hypothesis self-correction, is itself not subject to the same correction, as there is no rule to tell us just when the determinism hypothesis breaks down, no negative test of the hypothesis. The rule of practice, as part and parcel of the method itself, is that the hypothesis never does break down; it is the unsatisfactory prediction itself that motivates the search for causation, Newton pondering the fallen apple. The final justification of the method rests, as always, in the likelihood of future discovery.

It is in the event of unknown and thus possibly nonexistent physical laws that the program may run afoul the prevailing "belief that natural science, based on observation, comprises the whole of human knowledge", to quote from the Philosophy Pages entry for Positivism [26], where in the extreme view further elaborated, whatever the rational appeal or past successes of the determinism hypothesis, non-empirical statements of all brands are metaphysical [27]. Upon this reasoning an epistemically undetermined microscopic experimental outcome becomes, in accordance with von Neumann's reduction axiom and corroborating Copenhagen interpretation of QM, ontologically indeterminate. But more on this later. It should at least be clear

that a probabilistic theory understood also as complete (in respect of its account of the physically objective world) such that empirical collective statistics at the same time exhaustively characterize each collective-member also - such a theory naturally assumes a strongly subjective (e.g. Bayesian) quality.

In our lead-up to the logical no-go derivation (whose standard presentation is couched in a specialized nomenclature), we first, in the next section introduce the necessary experimental and formal terminology by way of considering an application of the QM probability theory to a specific instance.

2.1 the structure of experimental outcomes

Let us consider a physical system and the set of experiments that may be performed on it. To each experiment there corresponds an array of characteristic outcomes, an experimental spectrum, which for a sufficiently large number of identical experimental trials may then be mapped to a probability distribution, a state space, each element of which being equal to the long-run relative frequency recorded for the corresponding outcome. We consider the complete set of such distinct experimental processes. To the compound mapping then there corresponds a parallel mapping from experimental propositions (as we have seen, corresponding to outcome 'a', the proposition: 'the outcome is a') to the interval $[0,1]$, taken as a measure of the truth of a given proposition: mapped to '1' for true, to '0' for not-true. And like its experimental counterpart, this mapping too is generally non-injective [28], as distinct experimental arrangements may sometimes yield identical results; i.e., outcomes sometimes overlap (Classically, e.g., the 'weight' measurement outcome for a given mass on earth will be identical to the weight measurement, say on the moon, of an entirely different mass.).

The sort of quantum experimental data that readily lends itself to this description is obtained from measurements of microscopic spin of the kind taken in Stern-Gerlach (SG) experiments [29, 30]. There, an assemblage, or ensemble, of identically

prepared particles is accelerated through a localized inhomogeneous magnetic field from which they emerge with velocities in one of a discrete number of directions

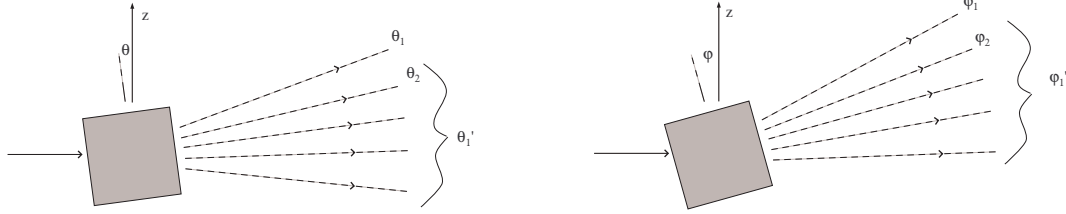


figure 1

a given direction characteristic of a particle's spin projection along the SG symmetry axis. We know however from experience with ordinary macroscopic spins and from the predictions of classical electromagnetism that these directions should instead vary continuously

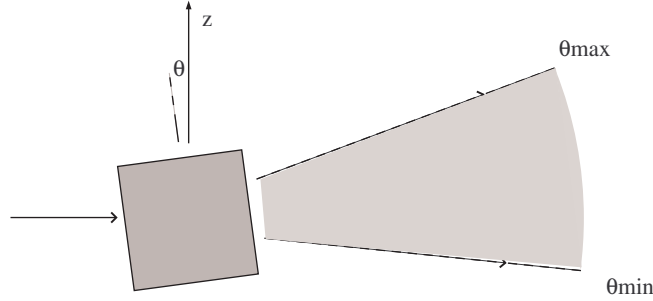


figure 2

with limits determined by the spin magnitude and SG field strength. Microscopic spins predictions are for this reason said to be 'quantized', appearing, observed, only in discrete amounts, and a SG experiment θ is thus characterized by its *discrete* outcome set $\Omega_\theta = \{\theta_1, \theta_2, \theta_3, \dots, \theta_n\}$, with outcome probabilities given by the experimental relative frequencies

$$P(\theta_i) = p_i = n_i/n$$

$$\sum n_i = n, \quad \text{so that,} \quad \sum p_i = 1$$

where n_i is the number of experimental trials with outcome θ_i , and n the total number of trials in a given run [29].

In respect of the formal probability space $(\Omega_\theta, \mathbf{F}, P)$, the probability measure P maps \mathbf{F} to the reals, $P: \mathbf{F} \rightarrow [1,0]$, where \mathbf{F} is the sigma algebra generated by Ω_θ , and is thus composed of the closed unions of subsets of Ω_θ , $E_i \cup E_j$, called events, where $E \subseteq \Omega$. Events then are sigma-measurable subsets and may always be expanded as a finite union of outcomes

$$E = \{\theta_i\} \cup \{\theta_j\} \cup \{\theta_k\} \dots = \{\theta_i, \theta_j, \theta_k, \dots\}$$

for which expansion we use the notation

$$E = \theta_i \cup \theta_j \cup \theta_k \dots \quad (1)$$

Elements of \mathbf{F} are called the measurable or Borel sets pertaining to the given experiment, while the probability measure P has the property $P(E_i \cup E_j) = P(E_i) + P(E_j)$ whenever events E_i and E_j are disjoint, denoted, $E_i \perp E_j$ [31]. Then with

$$\mathbf{F} = \{\theta_1, \theta_2, \theta_3, \dots, \theta_n, \theta_1 \cup \theta_2, \theta_1 \cup \theta_3, \dots, \theta_i \cup \theta_j \cup \theta_k, \dots, \theta_1 \cup \theta_2 \cup \dots \cup \theta_n\}$$

single element events, here θ_1 , θ_2 , and θ_3 , represent individual experimental outcomes and are said to be atomic or elementary; they are the *primitive* elements of the probability theory, external inputs of a truth value status independent of the theory, while the general \mathbf{F} element represents combinations of individual outcomes. As an experimental probability mapping is characteristic of the corresponding ensemble of observations, distinct formal probability functions P may be taken to represent distinct states of the ensemble.

It is possible to generalize the outcome set by taking at once the union of all outcome sets, $\Omega_\theta \rightarrow \Omega_\theta \cup \Omega_\varphi \cup \Omega_\chi \cup \dots = X$, called the outcome space [16]⁴.

$$\Omega \rightarrow X = \{\theta_1, \theta_2, \theta_3, \dots, \theta_n, \varphi_1, \varphi_2, \varphi_3, \dots, \varphi_n, \chi_1, \chi_2, \chi_3, \dots, \chi_n, \dots\} \quad (2)$$

⁴One might well question whether the criterion of "generalization" is met here. More on this later

A peculiarity of measurements on microscopic ensembles is the absence of experimental mappings $P: X \rightarrow \{1,0\}$ such that all ensemble members have, simultaneously, all the same projections. The phenomena is called dispersion; thus, all microscopic ensembles are observed to be dispersive [5].

2.2 formal structures in a Hilbert space

It happens that the forgoing formal relations are structured in a manner similar to those among the elements in a vector space. We consider then an n -dimensional Hilbert space (H-space) spanned by the representative basis

$$\Omega = \{|1\rangle, |2\rangle, |3\rangle, \dots, |n\rangle\}$$

The span of this basis, comprised of all possible linear combinations, $\alpha_1|1\rangle + \alpha_2|2\rangle + \alpha_3|3\rangle + \dots + \alpha_n|n\rangle$, where the α_i are complex numbers, constitutes the H-space itself. Among the basis elements are the structural relations [32]

$$\begin{aligned} \langle i|j\rangle &= \delta_{ij} && \text{(orthonormalization)} \\ \sum |i\rangle\langle i| &= \mathcal{I}_{n \times n} && \text{(completeness)} \end{aligned}$$

by means of which one orthonormal basis is related to another: $|j'\rangle = \mathbf{I}_{n \times n}|j'\rangle = (\sum |i\rangle\langle i|)|j'\rangle = \sum \langle i|j'\rangle|i\rangle = \sum c_{j'i}|i\rangle$. To each unit element then there corresponds a characteristic operator that projects any H-space vector $|\psi\rangle$ onto and so defines a unique subspace, $(|k\rangle\langle k|)|\psi\rangle = c_{k\psi}|k\rangle$, for some $c_{k\psi} < 1$. The operator $P_k = |k\rangle\langle k|$, thus projects an arbitrary vector onto the H_k subspace $\{|k\rangle\}$, and is known as a *projection operator*

$$P_k|\psi\rangle = c_{k\psi}|k\rangle.$$

The complete H-space is then a formal union of such subspaces

$$\begin{aligned} H &= H_1 \cup H_1 \cup H_2 \cup H_3 \cup \dots H_n \\ &= H_i \cup H'_i \end{aligned}$$

where H'_i here is the H space relative complement [48] to the H_i subspace.

2.3 semantical rules

When we now assign an experimental outcome set to an orthonormal basis, $\Omega \sim \mathbf{\Omega}$, and thus $X \sim H$, we identify a pre-measured state such as ψ in figure1 with an expanded vector in this basis

$$\begin{aligned} \psi &\sim |\psi\rangle = \sum c_{i\psi} |i\rangle \\ \sum c_{i\psi}^2 &= 1 \end{aligned} \tag{3}$$

and obtain the experimental statistics, the observed distribution, $P: \Omega \rightarrow \{p_i\}$, by means of the scalar product $\langle \psi | j \rangle$ as

$$p_i = |\langle \psi | i \rangle|^2.$$

Further, the observed ensemble dispersion manifests here as the nonexistence of H-space vectors $|\phi\rangle$ having the property

$$\langle n' | \phi \rangle = 0 \text{ or } 1, \quad \text{for all } n'$$

In other words, there can be no probability measure, no state, with the property, $P_\psi: H \rightarrow \{0, 1\}$.

In terms of projectors, the previous H-space structure relations become

$$\begin{aligned} P_i P_j &= \delta_{ij} P_j && (\text{orthonormalization}) \\ \sum P_i &= \mathbf{I}_{n \times n} && (\text{completeness}) \\ p_i &= |\langle \psi | P_i | \psi \rangle| \end{aligned}$$

The main advantage of this formulation lies in the correspondence between projection operators and experimental propositions. The projectors are QM operators with eigenvalue set $\{0, 1\}$, so that as a projector corresponds to an experimental proposition, $(\theta_i \sim |i\rangle \sim P_i)$, its eigenvalue corresponds to the proposition's truth value: '1' for 'true', '0' for 'false'; likewise, as the probability p_i gives the projection

of $|\psi\rangle$ along $|i\rangle$, the corresponding projector maps to the proposition θ_i , the proposition that $P_\psi \subseteq P_i(H_\psi \subseteq H_i)$. As a consequence, in this vector-*space* formulation of states we have that

$$P_i \subseteq \mathbf{I}_j(H_i \subseteq H_j \bigcup H'_j), \quad \text{for all } \theta_i \text{ and } \varphi_j$$

whereas in a vector-*set* formulation we have, as in set theory, $\theta_i \subseteq (\varphi_j \bigcup \varphi'_j)$, only in the event that either $\theta_i \subseteq \varphi_j$ or $\theta_i \subseteq \varphi'_j$.

2.4 logical structure of micro-events

By the logical structure of microscopic events we refer the interrelations among the propositions that assert the occurrence of such events. And as to each individual experimental outcome there is assigned a yes-no probability distribution, to the corresponding proposition is assigned a truth-value distribution, the two distributions, presumably, being one and the same.

Among experimental propositions, and propositions in general, there are ordering relations of implication, such that the truth of one proposition may imply that of another. This relation is typically expressed in the notation of naive set theory as set inclusion, $\theta_i \subseteq \varphi_j$, here θ_i implying φ_j . Whereas an equivalence of propositions, $\theta_i = \varphi_j$, simply represents the combined orderings $\theta_i \subseteq \varphi_j$ and $\varphi_j \subseteq \theta_i$. Consider for example a case in which two volumes physically overlap, $V_a \subseteq V_b$, and the proposition a (b): the particle is in volume $V_{a(b)}$. It is then by self-evident tautology that, $a \subseteq b$, and the relation is said to be analytic. On the other hand, there are many relations among propositions, also empirical, such as may embody e.g. the observation of a physical regularity or law and do not involve tautology. For example, given propositions a: the object is released from a height h, and b: the object reaches the ground in t_h seconds, an ordering, $b \subseteq a$, might express an instance of Newton's law of gravity. Such relations as these are synthetic [33]. In both cases, a is said to be a lower bound of b in the ordering $a \subseteq b$. In the set theoretic notation, the conjunction and intersection of propositions $a \bigcup b$ and $a \bigcap b$, are then taken to be

greatest lower bound (glb) and lowest upper bound (lub) of 'a or b' and 'a and b', respectively, and are said to be true whenever 'a is true or b is true' and 'a is true and b is true'.

The complete set of propositions bearing on the experiments that may be performed on a given physical system constitute a proposition system with structural properties characteristic, presumably, of the physical system itself. As it happens, the truth structure of the conjunction of two propositions is all important to a derivation of the logical no-go theorem. In most analysis, the conjunction of *any* two experimental propositions is again an experimental proposition having a truth structure given by the following rule:

Let I be an index set and $\{a_i\}(i \in I)$ any subset of $L, a_i \in L$. Then there exists a proposition, denoted by $\bigcap_I a_i$ with the property

$$x \subseteq a_i \text{ for all } i \in I \leftrightarrow x \subseteq \bigcap_I a_i$$

"axiom II" as it appears in Jauch's QM text [17]; the proposition system that satisfies this rule is then shown to have the structure of a mathematical lattice [14, 17, 18, 19] with

$$\begin{aligned} a &\subseteq a \text{ for all } a \in L ; \\ a &\subseteq b \text{ and } b \subseteq a \text{ implies } a = b ; \\ a &\subseteq b \text{ and } b \subseteq c \text{ implies } a \subseteq c. \end{aligned}$$

To every $a \in L$ there exists another proposition $a' \in L$ with

$$\begin{aligned} (a')' &= a ; \\ a' \cap a &= \emptyset ; \\ a &\subseteq b \leftrightarrow b' \subseteq a'. \end{aligned}$$

The axiom is assumed valid for experimental propositions in respect of both ordinary macroscopic and microscopic systems [1]. What sets one type apart from the other are the ordering relations between propositions that bear on experiments that do not commute, compound measurements for which the temporal order of component application has an effect on the eventual component outcomes. For example, if on a single physical system we perform the experimental sequence, $\theta\varphi\theta$, resulting in θ outcomes θ_i and θ_j that are not equal, $i \neq j$, then experiments θ and φ , and corresponding propositions, do not commute and are said to be incompatible. While the noncommutivity of measurements on microscopic systems is readily observed, the term 'classical', sometimes ascribed to macroscopic systems, refer properly, rather, very specifically to measurements, experiments, that commute: classical system \sim commutative measurements on system.

2.5 classical versus quantum logical structures

The truth structure of classical syntactically compound experimental propositions is given by implicit set theoretic rules such as the law of distribution

$$\theta_i \cap (\theta_j \cup \theta'_j) = (\theta_i \cap \theta_j) \cup (\theta_i \cap \theta'_j) \quad (4)$$

nicely illustrated by means of Venn diagrams

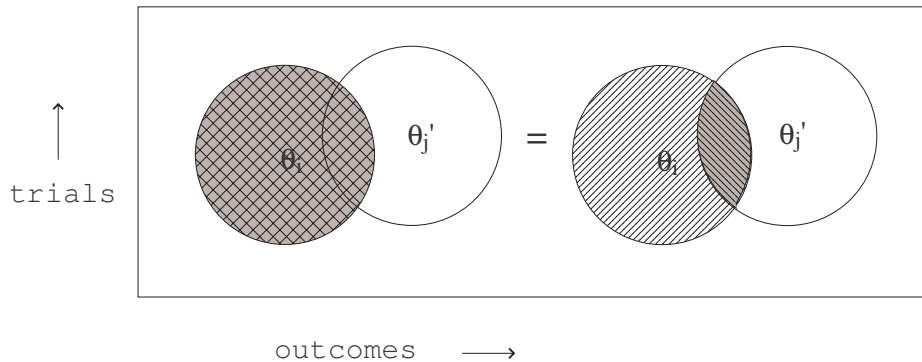


figure 3

where the shaded spatial areas are correlated to set size, hence to probability. A system of propositions obeying relation (4) is said to be Boolean (or classical) [34].

It was the mathematician and pioneering quantum theorist Von Neumann who long ago first observed that mutually noncommuting propositions generally do not satisfy the relation [35]. Thus, propositional systems that refer to classical phenomena are distributive, while those that refer to microscopic phenomena are nondistributive. Given the significance of this distinction, it is worth taking a close look at Von Neumann's argument as it appears in his *The Logic of Quantum Mechanics* [35]: "...These facts suggest that the distributive law *may* break down in quantum mechanics. That it *does* break down is shown by the fact that if a denotes the experimental observation of a wave-packet φ on one side of a plane in ordinary space, \acute{a} correspondingly the observation of φ on the other side, and b the observation of φ in a state symmetric about the plane, then (as one can readily check)"

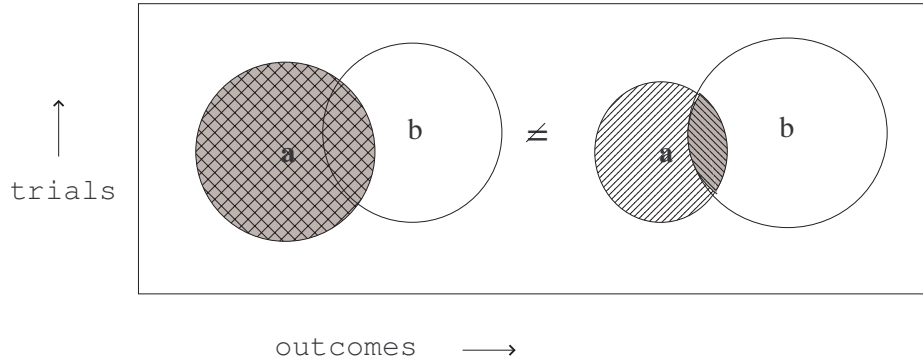


figure 4

$$a \cap (b \cup b') \neq (a \cap b) \cup (a \cap b') \quad (5)$$

since, actually, $(a \cap b) = 0$ when a and b are non-collinear. Another version of the argument in terms of spin measurement appears in Jammer's *Philosophy of Quan-*

tum Mechanics [36]. From this we see that while the logical matrix of microscopic phenomena affirms the law of the excluded middle according to which the proposition, $\mathcal{I} = a \cup a'$ (a or not- a), the proposition of 'identity', is always true, it denies the law of bivalence by virtue of which exactly one of the propositions ' a ' or ' a' ' is of necessity true [37].

2.6 logical no-go theorem

With the necessary machinery now in place it is here that we encounter a possible conflict with the notion common to realist thinking that to experimental processes there are causes that determine their outcomes with certainty⁵; here, the beginnings to the logical no-go. By means of this determinism experimental outcomes may in principle always be known prior to measurement, so that future tensed propositions are at all times bivalent, either true or false; hence, the realist principle of value-definiteness [36, 10, 39]. But the set of all microscopic ensemble measurements, as we have seen, is empirically dispersive

$$\sigma(a) \equiv P(a) - P^2(a) \neq 0 \quad \text{for at least some propositions } a$$

which then casts the realist ensemble as an assemblage of similarly prepared though non-identical entities, as a 'mixture' of dispersion-free sub-ensembles whose measurement yet yield the necessary (observed) noncommutivity of incompatible observables. Let us point out that this realist view contrasts the previously given 'conventional' view where ensemble dispersion appears rather as a direct manifestation of non-bivalent experimental values possessed not only by the assemblage, but by its individual members also. In any event, realist dispersive ensemble states, ω , are then linear sums of nondispersive sub-ensemble states, ω_i .

$$\omega = \sum \alpha_i \omega_i \quad \text{for some complex numbers } \alpha_i$$

⁵By realism we mean simply the realism e.g. characteristic of the EPR elements of reality [38, 36] which presupposes determinism as a sufficient condition [9], characteristic also of observables in Bell's theorem

with, for each ω_i

$$\omega_i(a) - \omega_i^2(a) = 0 \quad \text{for all propositions } a$$

together forming a convex set [16, 17, 40]. From this constraint on realist nondispersive subensembles, $\omega_i(a) = 1$ or 0, for all propositions a , it is easy to show by direct substitution into (4) that every definite truth-value combination corresponding to the set of definite values possessed by a given subensemble affirms the law of distribution. I.e., given propositions a and b and every possible definite value assignment, $a, b \in \{0,1\}$, we find for each case

$$a \cap (b \cup b') = (a \cap b) \cup (a \cap b')$$

The relation holds, recall, only in the case that measurements corresponding to constituent propositions commute. Given that microscopic measurements generally do not commute the realist hv interpretation of QM and its description of the microscopic data is placed at direct odds with observation, thus concluding the logical no-go proof.

$$\begin{aligned} \text{realist interpretation} &\Rightarrow \text{value-definiteness} \Rightarrow \text{dispersion-free mixtures} \Rightarrow \\ &\text{distributive logic} \Rightarrow \text{commutative} \Rightarrow \text{classical.} \end{aligned}$$

In the words of its authors, "Loosely stated, the main result [of the theorem] is simply this: if there exist incompatible observables then hidden variables are not possible." Notwithstanding critical appraisals from Bohm and later Bell (and sadly, few others), which we discuss in a later section, at the time of publication this pronouncement on hidden variables enjoyed unanimous approval.

3 physical and logical conjunctions

The inadmissibility of realist hv's on the grounds of non-classical commutivity is deduced it might have been noticed from no more than a glancing account of the

phenomena of noncommutivity itself; our fig 4, for instance, which illustrates the argument connecting noncommutation with nondistributivity advanced by von Neumann depicts no explicit noncommutation.

3.1 joint probabilities

The noncommutivity of measurements is a phenomena that necessarily involves the application of consecutive experiments *on individual physical systems*. An instance of this we denote by the inequation, $\varphi_j \theta_i \neq \theta_i \varphi_j$, where the spatial order of propositions here, right to left, represents the temporal order of experiment application. In words, the proposition that the compound experimental outcome is a then b generally has a different truth value from the proposition that the outcome is b then a . For comparison with fig 1 we illustrate an experimental arrangement for which noncommutivity effects are common.

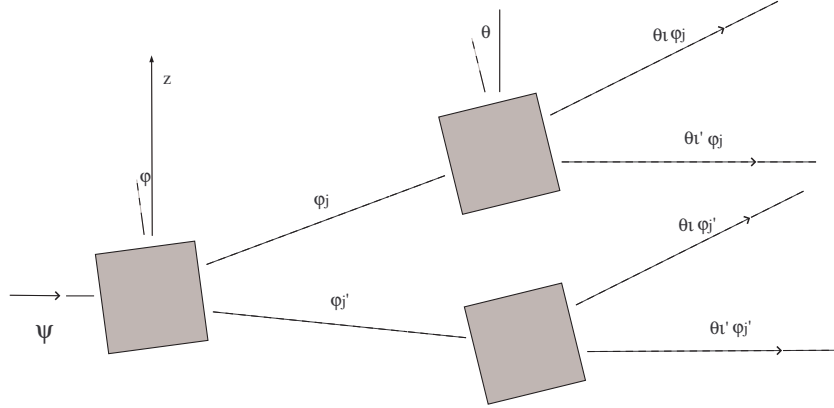


figure 5

Here the experimental outcome space may be given as the complete set of possible single SG measurement outcome combinations $\{\theta_i \varphi_j, \theta_i \varphi_j', \theta_i' \varphi_j, \theta_i' \varphi_j'\}$ (or as the set $\{\varphi_j, \varphi_j'\}$ in terms of marginal outcomes) whose probabilities contribute to the marginal probability

$$P(\varphi_j) = P(\theta_i \varphi_j) + P(\theta_i' \varphi_j) = P(\varphi_j | \theta_i) / P(\theta_i) + P(\varphi_j | \theta_i') / P(\theta_i')$$

The above expression $P(\alpha|\beta)$ here represents a conditional probability: the truth value of proposition α on the condition that proposition β *with regard to the same physical system* has truth value 1 [41]. An exchange of labels in figure 5, $\theta \leftrightarrow \varphi$, obtains the converse marginal probability

$$P(\theta_i) = P(\varphi_j\theta_i) + P(\varphi'_j\theta_i) = P(\theta_i|\varphi_j)/P(\varphi_j) + P(\theta_i|\varphi'_j)/P(\varphi'_j). \quad (6)$$

With these definitions quantum mechanics predicts and experiment confirms that in general

$$P(\varphi_j|\theta_i)/P(\theta_i) \neq P(\theta_i|\varphi_j)/P(\varphi_j). \quad (7)$$

And yet from the same probability calculus we have the identity

$$P(\varphi_j|\theta_i)/P(\theta_i) = P(\theta_i|\varphi_j)/P(\varphi_j) \equiv P(\theta_i \cap \varphi_j). \quad (8)$$

in which the expression to the far right is known as the *joint probability* for propositions θ_i and φ_j . The discrepancy between (7) and (8) is an expression of the well know fact that joint probabilities, by definition symmetric, $\theta_i \cap \varphi_j = \varphi_j \cap \theta_i$, *do not exist* for mutually noncommuting experimental arrangements⁶; the experimental determination of one distribution fundamentally disturbs the other [20, 21, 22, 23, 24], a case of which illustrated here Venn diagrams

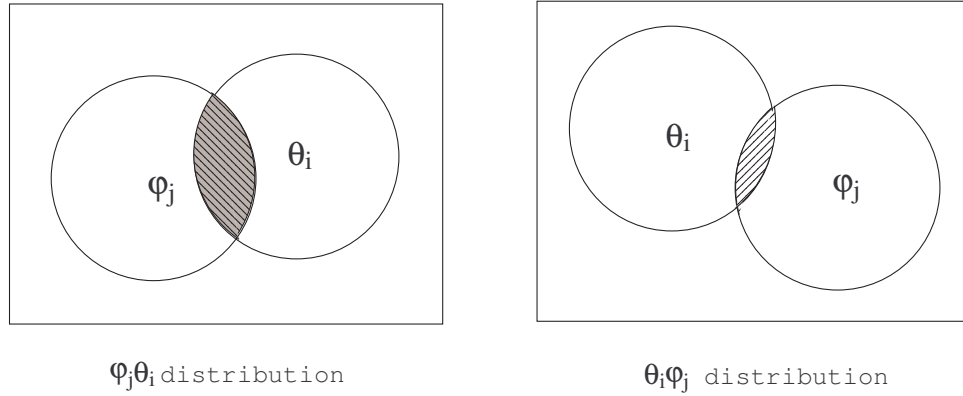


figure 6

⁶for a discussion in terms of corresponding random variables see ref. [42]

so that $\varphi_j\theta_i \neq \theta_i\varphi_j$, propositions θ_i and φ_j noncommuting.

3.2 nondistribution of probability

There is thus an ambiguity when signifying the conditional probabilities of non-commuting propositions in the usual set theoretic *notation* . The notation is also standard to classical logic whose matrix of propositions, like that of naive set theory, lacks the temporal ordering of events needed to account for noncommutivity. Martin Strauss, a natural philosopher who has written extensively on the subject of QM interpretation, long ago made the point that, "Classical probability based on the usual propositional calculus which is isomorphic to the set-theoretic system of subsets of a given set has for its probability functions a domain which is likewise isomorphic to the system, and *assumes* therefore the simultaneous decidability of any two propositions...", the quote taken from Jammer [36]. In a more suitable though unfortunately less common notation we have that $P(\varphi_j|\theta_i)/P(\theta_i) \equiv P(\theta_i, \varphi_j)$ [43], so that

$$P(\theta_i, \varphi_j) = P(\varphi_j, \theta_i) \equiv P(\theta_i \bigcap \varphi'_j), \quad \text{only in the event that } \theta_i\varphi_j = \varphi_j\theta_i.$$

On the other hand, from Eqn (6) we have the always valid identity

$$P(\theta_i) = P(\varphi_j, \theta_i) + P(\varphi'_j, \theta_i)$$

although, again, generally

$$P(\theta_i) \neq P(\theta_i \bigcap \varphi_j) + P(\theta_i \bigcap \varphi'_j). \quad (9)$$

Relation (9) states a nondistribution, a violation of the usual distribution of relative probabilities. As the noncommuting conjunction itself, $\theta_i \bigcap \varphi_j$, does not exist as an experimental proposition [21, 23, 24], the nondistribution is based ontologically.

3.3 syntactic and semantic conjunction

In the nondistributive relation of the preceding section, Eqn. (5), the conjunction appears as a syntactic construct, an element in the object language of an X-space generated sigma-algebra, whereas above in (9) the conjunction corresponds to an atomic event and thus enjoys the status of a primitive whose semantics derive entirely within the meta-language of the probability theory [36, 44]. As classical probability spaces are induced by their corresponding experimental outcome sets [42], the former syntactic conjunction belongs to one probability space (an $H \cup H'$ subspace), the latter semantic conjunction to another (product of subspaces, $H \times H'$, itself however not a product space [17, 45]). This unfortunate formal mis-identification has not helped resolve confusions in an already contentious QM interpretations debate [46]. Let us be especially clear on the point we have just made: It is the syntactic nondistributivity of propositions, equation (5), that characterizes classical or Boolean logic,

$$\text{syntactically distributive logic} \Rightarrow \text{classical logic}$$

whereas the semantic nondistributivity of probabilistic measurements, equation (9), is characteristic of noncommutative or 'classical' probability,

$$\text{semantically distributive probability} \Rightarrow \text{classical probability.}$$

Whether the two relations express the same physics is a question taken up in the next section.

3.4 philosophical differences

Several adherents to the conventional interpretation such as von Neumann, Piron, and Jammer have offered specific examples of the physics behind the noncommutative conjunction. Bohm, Bub and Bell in response maintain by means of other semantics that such a conjunction does not exist, that there can be no physical correspondence: To quote Bohm "...a and b, represented by noncommuting projection

operators, can both be true with certainty if they are confirmed as such by corresponding processes, whereas (because of interference) *no process exists* to verify the proposition $a \cap b$. In this case $\omega(a \cap b) = 0$ without excluding the possibility $\omega(a) = \omega(b) = 1$ ” [21]. On the other hand, it appears to be clear that upon the criteria set down by von Neumann and others, the conditions, $\omega(a) = \omega(b) = 1$, have the very meaning, $\omega(a \cap b) = 1$... And then from Bell, ”We are not dealing in B [a system of experimental propositions] with logical propositions, but with measurements involving for example differently oriented magnets. The axiom [if $\langle a \rangle = \langle b \rangle = 1$, then $\langle a \cap b \rangle = 1$] holds for quantum mechanical states. But it is a quite peculiar property of them and in no way a necessity of thought”. Both criticisms appearing within the context of their respective refutations of the logical no-go theorem are it seems well-founded, though they might have been better, more effectively, put within the context of a larger, more comprehensive assessment of the logical no-go theorem, within the context of a kind of critical analysis that of the Bell’s theorem, e.g., may be found everywhere [8, 9] ⁷.

⁷Bohm might have added within this context ’that propositions cannot be decided upon simultaneously (as with the incompatible semantic conjunction) does not impose that they are simultaneously undecidable (as in the case of the incompatible syntactic conjunction)’, or Bell, that ’the impossibility of simultaneous incompatible measurements (as in the case of the incompatible semantic conjunction, $\langle a \cap b \rangle = 0$) does not preclude the simultaneous possibility of incompatible measurements (as in the case of the incompatible syntactic conjunction $\langle a \rangle, \langle b \rangle \neq 0$)’ . And so forth. As it happened, each man instead wage direct assaults on the notion of realist semantic distributivity without shedding much light on the seeming validity of syntactic distributivity, i.e. without resolving the confusion between the two relations, a confusion that persists to this day. The same may be said in respect of Strauss’s ’complementary logic’ [23] and Suppes’s later version, both of which effectively make the point though without an adequate appreciation for the distinction between compound and elementary ’events’, syntactic and semantic conjunction.... and consequently confounding the notion of the ’classical’ - of classical logic with classical physics. This, from the syntactic rule of Strauss (long before the work of Bohm and Bell) to Gudder’s restriction to *experimental questions* [22] and several analysis that have appeared since. It comes as little surprise that with few exceptions it is syntactic distributivity that holds the interest of mathematics and philosophy (as evidenced in the works of mathematicians and philosophers), while the

The logical no-go theorem mistakes syntactic distributivity for semantic distributivity and thus confounds the realist notion of deterministically possessed values with the notion of the commutation of incompatible experiments. While the two are certainly conceptually distinct, there remains the possibility that the relations are empirically related.

4 reconsideration of the argument

Then what of the syntactic nondistributivity at the heart of the logical no-go theorem? What does it signify, and what, physically, would constitute an instance of it; how might such an instance be confirmed or refuted? The answer to these questions, as anticipated in the previous section, supervenes on the precise semantics associated with the noncommutative conjunction, for which, as initial guidance we may take the examples offered separately by von Neumann, Piron and Jammer.

4.1 syntactic and semantic physical distinction

We consider an experiment in which the incompatible observables, a and b , are randomly sampled over an ensemble of 'identical' systems - an ensemble. Then, according to our three authors, given a sufficiently large sample we should never find that both $P(a) = 0$ or 1 and $P(b) = 0$ or 1 . I.e., we should never find that for each proposition, a and b , the individual measurement outcomes are all identical, either all yes, or all no. The equation, $P(a \cap b) = 0$, thus states an instance of this, of the impossibility of finding the ensemble simultaneously in eigenstates of propositions a and b . But clearly, this is identical to the condition of dispersion itself, and furthermore does in no obvious way speak to the relevant question of measurement noncommutivity. Might these simple points have escaped their notice.

empirically inclined experimentalists have concerned themselves primarily with the semantic, as only the semantic noncommutative conjunction meets the criteria to describe physical events (or non-events) [47]

Probably not. It is more likely that our authors thought it reasonable to require of realist *outcome* states the same statistics as QM eigenstates. We illustrate the assumption for nondispersive state ψ with truth value 1 for experimental proposition θ_i (and also φ_j by simply making the exchanges, $\theta \leftrightarrow \varphi$ and $i \leftrightarrow j$)

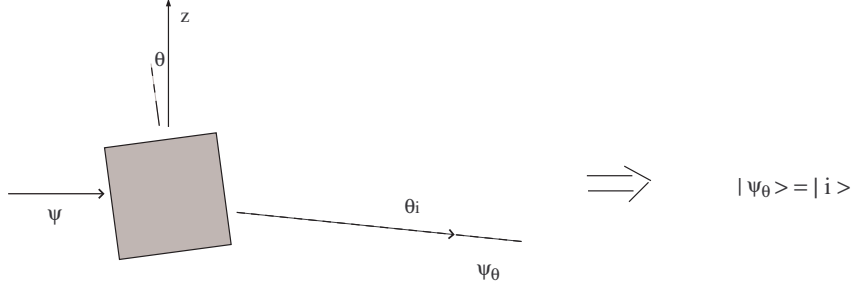


figure 7

Then, $P_j P_i |\psi\rangle = P_j |i\rangle$ and $P_i P_j |\psi\rangle = P_i |j\rangle$, from which we have the probability relation, $P(\theta_i \varphi_j) = P(\varphi_j \theta_i)$, same as for the case of QM states under the operation of mutually commuting experiments. There is however no reason a priori to require such statistics of subensembles of dispersion-free states, and it is easy to construct an empirically consistent measurement picture in which they would not hold, as the time evolution of a nondispersive state may depend, very reasonably, upon the measurements performed on the representative system. Then by parameters $\lambda_{\theta\varphi}$ and $\lambda_{\varphi\theta}$ let us denote this dependence in the case of two noncommutative temporal sequences of experiments θ and φ , and relate the state of the system prior to measurement, $\psi(t)$, to its state following measurement, $\psi(t')$, by means of a time evolution operator U which propagates the state vector from t to t' : $\psi(t) \rightarrow \psi(t') = U(\lambda_{\theta\varphi}; t', t) \psi(t)$, $U(\lambda_{\varphi\theta}; t', t) \psi(t)$. These then describe the two distinct subensemble experimental processes

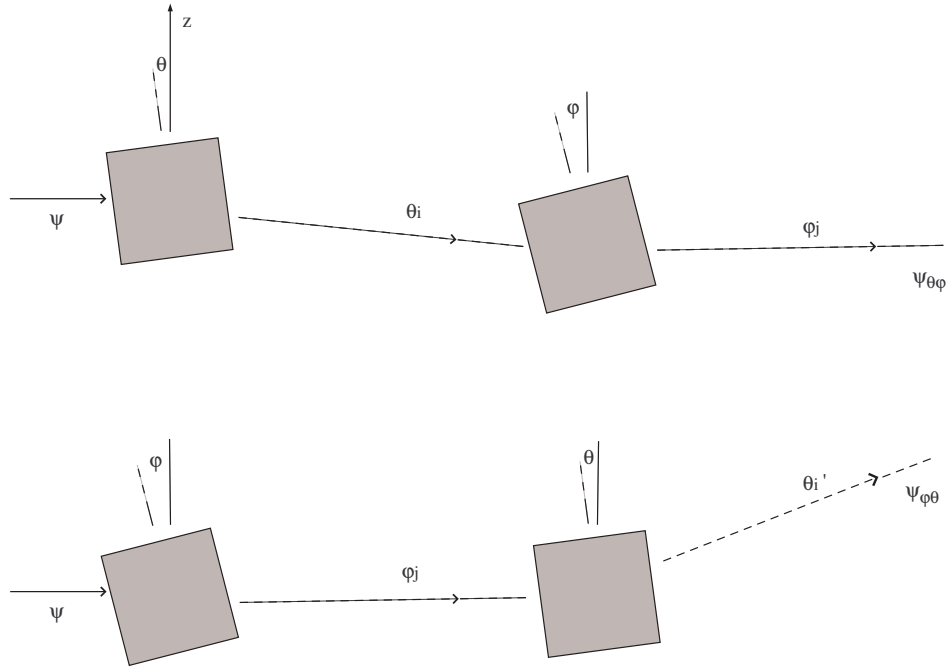


figure 8

So that while each initial subensemble is nondispersive, $\omega_{1,2}(\theta_i), \omega_{1,2}(\varphi_j) = 1$ or 0 (though with a combined non-zero dispersion: $\omega(\theta_i), \omega(\varphi_j) \neq 1$ or 0), in violation of syntactic nondistributivity, eqn.(5), the consecutive measurement of incompatible propositions here maintain noncommutivity, $P(\theta_i \varphi_j) \neq P(\varphi_j \theta_i)$, thus satisfying the constraints of semantic nondistributivity, eqn. (9), the one condition thus independent of the other.

In addition, this constraint linking nondistributivity to noncommutivity and necessary to the logical no-go theorem is not amenable to empirical testing, as the nondispersive ensembles themselves exist solely as hypotheticals, within abstract partitions of the physical dispersive ensembles of experience. To again quote Bell [20], there is 'no necessity of thought' by which dispersionless subensembles must obey the statistics of QM, none then leading from nondispersion to commutivity, a violation of one not infringing upon the possible validity of the other. We may

well accept then the existence of syntactically distributive, hence nondispersive, subensembles without contravening their semantic nondistributivity, i.e. the non-commutativity of incompatible measurements performed on all subensembles. This assertion, let us be clear, is at odds with the claim made by Jauch and Piron in Ref.[14] that 'The detailed analysis of this relation [syntactic distributivity] shows that it has exactly the properties one would associate with measurements which can be performed simultaneously without disturbing each other [noncommutivity].', which continues, 'For instance, if the propositions are represented by projection operators in a Hilbert space '. As it turns out, the cited "instance" serves also as the sole 'detailed analysis' ever offered or referenced by the authors in support of the claim, and it seems indeed the only one available. Thus, the statistical constraint implicit to the logical no-go theorem and imposed on nondispersive states rests finally on the premise that all states are quantum mechanical and hence dispersive. The argument is remarkable in its blatant circularity: All states dispersive \rightarrow No states nondispersive. Unlike the hv's argument advanced by von Neumann, however, recognized only a full 30 years after publication as simple fallacy, this basic circularity in the logical no-go argument did not long go unnoticed and was at publication quickly pointed out (in another context) by Bohm [21, 42] among others.

4.2 formal refutation

To continue one more, this problem with the logical no-go may also be understood from a more formal standpoint, physical considerations aside. The law of distribution holds whenever its components are sets, though not when they are spaces, as formally defined. In fact, when we faithfully respect component status the nondistributivity inequality (5) is immediately made distributive. Generally, given any outcome space $X = \bigcup_{\alpha} \Omega_{\alpha} = \{a, b, c, d, e, f, g, \dots\}$:

$$\begin{aligned} a &= a \bigcap \mathbf{I} \\ &= a \bigcap (b \bigcup b') \end{aligned}$$

$$\begin{aligned}
&= (a \cap b) \cup (a \cap b') \\
&= (a \cap b) \cup [a \cap (a \cup c \cup d \cup e \cup f \cup g \cup h \cup \dots)] \\
&= (a \cap b) \cup a \cap (a \cup c \cup d \cup e \cup f \cup g \cup h \cup \dots) \\
&= a \cap [(a \cap a') \cup (b \cap b') \cup (c \cap c') \cup (d \cap d') \cup \dots] \\
&= (a \cap a) \cup (a \cap a') \cup (a \cap b) \cup (a \cap b') \cup (a \cap c) \cup (a \cap c') \cup (a \cap d) \cup (a \cap d') \cup \dots \\
&= a
\end{aligned} \tag{10}$$

where, $(a \cap x) = 0$ whenever, $a \neq x$.

Syntactic nondistribution formally proceeds then from a set theoretic inconsistency, where the compliment-defining universal set [48] on the rhs of (5) is outcome set $\Omega_b = \{b_1, b_2, b_3, \dots, b_n\}$, but on the lhs is taken to be the outcome space X , thus yielding the (5) inequality

$$a \cap (b \cup b'_X) \neq a \cap (b \cup b'_\Omega) = a \cap b + a \cap b'_\Omega = 0 + 0 \quad \text{whenever } a \cap \Omega_b = 0.$$

Physically, of course, $b \cup b'_X$ is not an experimental proposition.

Alternatively, we may assign the inconsistency to the connective conjunction itself, \cup , which joins two propositions on one side of (5) to give their (QM) span, while on the other their usual set theoretic (classical) union [49].

In addition to the critical analysis of Bohm, the philosopher Popper was also quick to weigh in, pointing out, in a lively exchange with the logical no-go authors an inconsistency in their reasoning [50]. Though certainly blatant ($(b \cup b') \rightarrow (b \cup b'_X) = (b \cup b')$), nothing more, writes Pooper, than 'a simple slip') the misrepresentation is obscured by its logical idiom. Now at a safe distance one may appreciate the extent to which this notational slip accommodates the interpretation of the Copenhagen school: As Ω here designates the set of possible outcomes of an experiment performed on an individual ensemble member, e.g. on an individual particle, their truth values in the realist interpretation bivalent, the outcome space X , on the other hand, as a union of such outcome sets, $\cup_n \Omega_n$, then designates the set of possible outcomes corresponding to groups of experiments performed upon groups

of individual systems, upon ensembles, their truth values in any interpretation generally non-bivalent. To substitute one for the other here would be to mistakenly impose a constraint existing within one interpretation, the orthodox interpretation, in the course of evaluating an opposing interpretation, the realist interpretation [42]... ; formal sets always satisfy relation (5), spaces generally do not, and the very possibility of a consistent realist interpretation is immediately ruled out when it is assumed that H-space vectors represent all possible physical states. And so, again, the circularity.

5 summary and conclusions

We have shown that the logical no-go theorem involves the fallacy which argues that the empirical validity of QM maintains only to the exclusion of other descriptions, specifically, to the exclusion of a presumably more complete realist hv theory, that because observed ensemble states are all quantum mechanical, so too must their constituent subensembles down to the single element be. The theorem assumes, in the words of Bohm, "that the current linguistic structure of QM is the only one that can be used correctly to describe the empirical facts underlying the theory" [21], but from Bell, "only QM averages over the dispersion free states need produce this [observed statistical] property."

5.1 differing views on probability theory

If one accepts the proposition that qm states and observables refer solely to ensembles and their averages - a proposition accepted, in fact, by all practicing experimentalists as operationally valid - there opens the possibility of the existence of a distinct underlying individual ensemble-member reality, much as the Newtonian reality of individual systems underlies the dynamics of statistical mechanics, as the individual molecular reality of mutual electromagnetic interactions underlies the ideal gas law,

etc. Such is a *frequentist* understanding of QM probabilities as nothing more than the relative frequencies of ensemble measurement outcomes. This is the view taken e.g. in Ballentine's statistical or *ensemble* interpretation of QM [51, 52] where the absence of an exhaustive description of individual ensemble member reality casts QM as a theory incomplete and possibly provisional, as an approximation to a more complete theory, again, much as statistical mechanics is incomplete with respect to Newtonian mechanics. This was also generally the view of QM statistics held by Einstein [52].

From an opposing point of view, QM probabilities refer rather to individual events, to the outcomes of individual measurements made upon individual ensemble members. In this case, empirical ensemble frequencies relate to the 'likelihoods' of particular individual outcomes. Thus, the non-bivalence ordinarily characteristic of ensemble propositions are immediately manifest in the propositions regarding measurements on individual ensemble members. And so, to refer back to the experiment of fig. 1, not only is the spin projection of a particle not known before it has been measured (epistemically non-existent), but at such a stage the particle does not properly possess a spin (ontologically non-existent); it may only be said that the particle possesses a kind of likelihood that a given projection outcome will be obtained upon measurement. Such is a subjective *Bayesian* view of probability [53, 54] and is more or less in line with the orthodox interpretation of QM as a statistical though physically complete theory.

When experiments share a common outcome, i.e., when their outcome sets overlap, the theory predicts and experiment verifies that the relative frequencies for those outcomes are equivalent, independent of experimental context. This peculiar *noncontextual* behavior of microscopic ensemble statistics thus becomes by interpretation characteristic also of individual microscopic measurements: propositionally, $\theta_i \cap \Omega_\theta = \varphi_j \cap \Omega_\varphi$ in the event that we have physically $\theta_i = \varphi_j$.

5.2 noncontextual probability space X

The conventional interpretation necessary to the logical no-go theorem is effectively axiomatized by means of the probability space transformation (2), $\{\Omega_n\} \rightarrow \bigcup_n \Omega_n = X$. Earlier we spoke of this as a "generalization" of the probability theory, but the term is not really appropriate here, as the loss of generality is indeed enormous; rather, the probability mapping is now *specialized* to the case of noncontextual outcomes. The famous theorem of Gleason [55] illustrates the point: Given the class of outcome sets Ω_n of rank > 3 , it is not possible while also respecting the empirical sum rules, $P(\Omega_n) = 1$, i. e., $P(\Omega_n \cap X) = 1$ for all Ω_n , to map the corresponding propositions to a bivalent state space, $P : X \rightarrow \{0,1\}$. It thus follows from the harmless identity beginning our equation (10)

$$a = a \bigcap (b \bigcup b')$$

that

$$P(a) = P[a \bigcap (b \bigcup b')] \quad \text{for all } a$$

constraining at least some of our propositions, $a \in \Omega_a$, to now non-bivalent truth values - a constraint, when individual measurement outcomes are assumed independent of experimental context, strictly enforced by Gleason's theorem. The probability space transformation, this generalization, affects also of course a corresponding shift in the logical form of the involved experimental propositions, a shift from the properly conditional propositions of experimental physics (with experimental contexts antecedently given or understood. E.g., a proposition a_i which says that an outcome a_i will be obtained *when* or *if* an Ω_a experiment is performed) to propositions that are categorical. Thus from propositions epistemic to those whose claims are ontological [56]⁸. When with this noncontextuality (which, again, constrains

⁸Classically, this amounts to taking weight rather than mass as an intrinsic property of a body. In practice, the intrinsic masses are indeed determined, typically, from weight measurements, though not without the law of gravity and a consideration of experimental context (mass of the planet on which the weight measurement is taken)

a propositional equivalence $\theta_i = \varphi_j$, whenever holds the physical outcome equivalence $\theta_i = \varphi_j$) is assumed in addition the principle of the value definiteness, $P(\theta_i), P(\varphi_j) \in \{0, 1\}$, which holds for realist dispersion-free states, we have that the outcome of a measurement θ_i taken in the experimental context θ is the same as it would have been had the measurement been taken instead in the experimental context φ

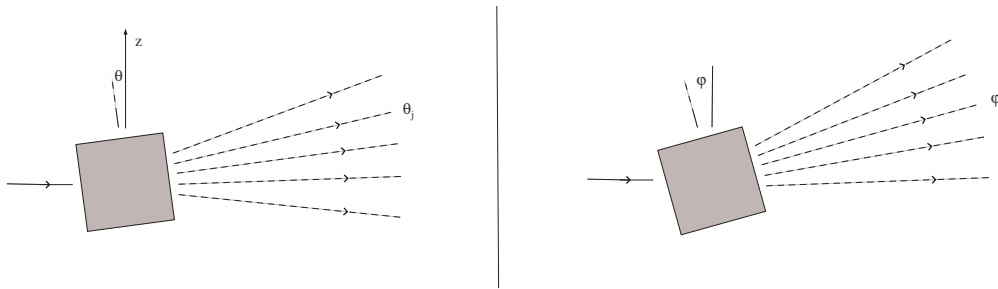


figure 9

a statement of the counterfactual definiteness of events, often understood under appropriate conditions as a formulation of determinism [57, 10].

This is also the constraint at the heart of the later and related no-go theorem of Kochen and Specker [58, 39, 10]. To briefly state, the KS analysis assumes as a necessary element to any consistent realist view the noncontextual embedding of QM observables in a nondispersive theory by means of the map, $P(\Omega_n \cap X) = 1$ for all Ω_n (respecting sum rules). But Gleason's already analytically rules out the possibility of such an embedding⁹. Interest in the KS analysis itself however persists

⁹It is remarkable that some, particularly mathematicians, seem at an utter loss why anyone would want to imagine a *contextual* embedding, $X \rightarrow \{\Omega_n\}$; physicists generally have less trouble. Remarkable that anyone, including mathematicians [59], would think a value-definite embedding of QM observables necessary to a realist hv interpretation, a realist reading that *begins*, after all, with the set of physical outcome sets $\{\Omega_n\}$. But the thinking is not so uncommon as one might

to this day, and its relevant issues are taken up by the writer in separate paper [10].

5.3 further speculations

In respect of the KS analysis, let us add here briefly one telling point of reference. While the system of micro-experimental propositions in the work of Jauch and Piron has a nondistributive, non-Boolean structure, and forms a complete orthocomplemented lattice, the QM logic of Kochen and Specker, on the other hand, is structured partially distributive, partially Boolean, and forms an orthocoherent orthoalgebra [16, 58]. Consider with this the existence of several well-known deterministic models of microscopic phenomena that also account for the noncommutation of incompatible observables (e.g., the famous model by Bohm [61] or a more recent one by Aerts [62]), of the presence of an elephant in the room, and it is obvious that it is not a question of the nondispersion or the noncommutivity of measurements that is at the heart of popular dissatisfaction with "classical" readings of QM; here, the 'logical' approach to the question of hv's misses the point. What is seminal to popular anti-classical sentiment in the case of QM interpretations and in the natural sciences generally is an enduring suspicion of the scientific enterprise that explains, predicts, and manipulates [63, 64] by virtue of the properly 'Classical' operational hypothesis according to which physical causes are by nature nomologically epistemic [65] hope. At least not so in the opinion of S. Goldstein [60]: " In view of the radical character of quantum philosophy, the arguments offered in support of it have been surprisingly weak. More remarkable still is the fact that it is not at all unusual, when it comes to quantum philosophy, to find the very best physicists and mathematicians making sharp emphatic claims, almost of a mathematical character, that are trivially false and profoundly ignorant." An exception are the views expressed by philosopher H. Stein who in 1970 writes, "There is no obvious way to interpret eventualities [QM projectors] as units of discourse, except under the conditions of a test realizing them (when an eventuality may be correlated with the proposition that *that eventuality holds*); but the eventualities realized together in any experiment have ordinary Boolean logical relations to one another, so that no non-standard logic here comes into play" [54]

- i.e., lawlike - and consequently deterministic ¹⁰. This, even against the stringent empiricism insisted upon by so many founders of the quantum theory. Such as that of W. Heisenberg [66], who was happy to evoke this kind argument in defense of a QM completeness under threat ¹¹, suggesting a certain determinism (excuse me, determination) on the part of the anti-realist, the anti-determinist camp. That this does indeed appear to be the central issue in dispute is rarely noted in the literature (See e.g. Ref.[22]), and on those occasions, as here, only in passing.

Of the three impossibility proofs we have mentioned only Bell's examines the hv idea on the relevant question of determinism; it is also the simplest of the three. Increasingly it is this theorem like the man that is the most interesting and compelling. A definitive test of the inequality however (notwithstanding a chorus of

¹⁰- a synonymy embedded in everyday usage: The official speaking on behalf of BP oil company in a news report on a recent refinery accident (24 March 05) assures the public that they will find out "why it took place, what were the causes".

¹¹What the student and non-specialist may find alarming - what alarms the writer - is the strength of *conviction* openly expressed in the QM foundations literature by parties on all sides of the interpretation issue.... In an otherwise excellent review, A. Fine writes "It should be clear by now that I find the instrumentalist interpretation of the theory repugnant. Indeed I find it sad that the discredited philosophical positivism of the 1930's, away from those doctrines the behavioral sciences are finally being weaned, should find its last ditch supporters among the middle generation of physicists..." [67]. I am inclined to agree; sad indeed..., but such intensity is perhaps less surprising when one considers the also uncharacteristic and much publicized early foundations debate between Einstein and Bohr [36]. Ballentine, under fire, maintaining his usual composure and clarity laments that "The entirely reasonable question, 'Are there hidden-variable theories consistent with quantum theory, and if so, what are their characteristics?,' has been unfortunately clouded by emotionalism. A discussion of the historical and psychological origins of this attitude would not be useful here. We shall only quote one example of an argument which is in no way extreme (inglis, 1961, p.4), 'Quantum mechanics is so broadly successful and convincing that the quest [for hv's] does not seem hopeful.' The vacuous characteristic of this argument should be apparent, for the success of quantum theory within its domain of definition (i.e., the calculation of statistical distributions of events) has no bearing on the existence of a broader theory (i.e., one which could predict individual events.)". [51]

claims to the contrary [68]) has yet to be run [10, 69].

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